Legal Reasoning and Some Logic after All the Lessons of the Elders

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When a philosopher crosses the border into another discipline, he is taking a risk. What motivates this visit? Was he invited, or is he just showing up on his own? […] when he crosses into the land of law; cui bono?

John Woods [80, p. 5]

Abstract

The passionate and staunch defence of logic of the controversial thinker Ibn Ḥazm of Córdoba, Abū Muḥammad ᶜAlī ibn Aḥmad ibn Saᶜīd (384-456/994-1064), had lasting consequences in the realm of legal reasoning. Indeed, the main aim of his book *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples (Kitāb al-Taqrīb li-ḥadd al-manṭiq wa-l-mudkhal ilayhi bi-l-alfāẓ al-ᶜāmmiyya wa-l-amthila al-fiqhiyya)*, composed in 1025-1029, is the study of methods of decision-making in legal contexts.

According to Ibn Ḥazm, if logic should have any role in real legal practice, then it should be built on the study of paradigmatic real cases of juridical decisions. In order to do so he undertakes a thorough study of deontic notions and their modal counterparts, that makes him one of the fathers of the logic of norms.

The basic units of Islamic deontic logic are what we might call, indulging in terminological anachronism, *heteronomous imperatives*. The point of *heteronomous imperatives* is to develop a logic of norms where the contentual analysis of deontic qualifications such as *Obligatory*, *Forbidden*, *Permissible*, *Facultative*, are put into practice, in order to justify transferring a juridical decision from a known case to an unknown one. Islamic deontic notions qualify the performance of actions as worthy of being *rewarded (in different degrees), sanctioned or neither*. In a more modern and general framework we might use the qualifications *law-abiding*, *law-breaking* and *legally neutral*

[0] Sections 1 and 2 of the present paper are based on [69].

(neither *law-abiding* nor *law-breaking*) instead — on the grounds of which the agent may be sanctioned or not. We claim that the logic of heteronomous imperatives that grows out from Ibn Ḥazm's insight, when shaped by Martin-Löfs Constructive Type Theory, suggests a way out of many of the paradoxes of deontic logic in legal contexts. Moreover, if we combine the logic of heteronomous imperatives with the Islamic argumentation theory for parallel reasoning, a system of legal reasoning results, that does not drop out the earth-bound human agent and is rooted in actual legal practice.

1 Introduction

When logic took the mathematical turn in the nineteenth century, the human reasoner dropped out of the picture, save (at most) as a highly idealized abstraction. Although much of present-day logic retains this indifference to the realities of human cognitive agency, there has of late been no want of effort to enrich the mathematical mechanisms of formal logic in hopes of achieving a tighter fit between theory and the reasoning-behaviour of the earth-bound human agent.

John Woods [81, p. 403]

1.1 Some general remarks

Since Kripke's [47] *Naming and Necessity*, it has been made public that early claims concerning the richness of possible-world semantics to express several forms of necessity have to be nuanced — at least in the case where propositional modal logic is extended with quantifiers.

Actually, already by the sixties, possible-world interpretations of deontic necessity, as developed by von Wright [77, 78], struggled with a wealth of philosophical and logical puzzles that threatened the framework right from the beginnings, and this already at the propositional level.[1] Despite the fact that, at a first sight, the notion of possible world, a counterfactual situation, seems to offer an appealing instrument to grasp the content of a normative statement prescribing how the world *should be*, it is by now apparent that standard truth-functional semantics underlying Kripke-style modalities has no direct way to deal with the dynamics required by the logic of actions and prescriptions.[2] Actions are in principle

[1] For recent overviews on those challenges see Hilpinen and McNamara [38] and Navarro and Rodríguez [63].

[2] *Dynamic Epistemic Logic* takes the dynamic challenge seriously — cf. van Ditmarsh et al [76]. However, it shares with *static* modal logic the meta-logical perspective on meaning. Accordingly, propositional effects of actions, truth-functional changes in the model, are described

not bearers of truth, and an appeal to possible-worlds does not explain by itself the incidence in the actual world of a prescription to act. After all, my cruising through red-traffic lights will be sanctioned in a state of the actual world after the infraction took place, not in a virtual possible world one. Of course, deontic necessity is a kind of necessity. It is just that standard model-theoretic semantics does not seem to be the right instrument to deal with the temporal dimension involved in the notion of norm, or important refinements are required.

It might be fair to say that possible-world semantics in general and contemporary standard deontic logic in particular is a late offspring of the propositional turn launched by the Stoics[3]. Indeed it were the Stoics, who under the background of a dynamic ontology constituted by events and actions, proposed to extend or perhaps even substitute Aristotle's relational approach to necessity with a propositional one,[4] whereby connectives and inference rules played the role of

at the metalevel. While in the framework of Dynamic Epistemic Logic, expressions for public announcements (i.e. assertions) are integrated into the object-language, their content, such as *proposition p is true*, is established at the meta-level (see [14]). It does not seem that in such kind of approaches prescriptions are first-class denizens of the domain of the actual world.

[3]This background might elucidate the historical roots of the predominance of the propositional perspective over the first-order one in contemporary main-stream Dynamic Epistemic logic — besides the technical problems concerning complexity constraints.

[4]Bénatouïl (2017) recalls that quite before Łukasiewicz [55] famous paper on the history of propositional logic, Brochard [17, 18] and Hamelin [32], not only acknowledged what we called the *propositional turn* but they also discussed if the Stoics proposal amounted to a replacement or rather to an extension of the Aristotelian metaphysical framework of essences with one rooted in physics and events — unfortunately, neither Brochard nor Hamelin seem to be acquainted with Frege's notion of concept as function (the origin of Russell's *propositional function*) relevant to the Stoic project of linking propositional and predicate logic. Deleuze [26, pp. 13–21], who does mention neither Brochard nor Hamelin in his chapter on the Stoics *Deuxième série de paradoxes, effets de surface*, on one hand praised them for moving away from Aristotelian essential predication, but on the other criticized them for reducing all to identity. In fact Deleuze [26] follows here Bréhier ([16], [16, reprint, p. 2 and p. 23]) who contests the "naturalistic" interpretation of Stoic logic by Brochard and Hamelin and who reads Stoic logic as the project of reducing all kinds of necessary links to identity — it seems that Bréhier and Deleuze, who follows the former, think that the Stoic notion of necessity reduces to logical necessity after all. In fact, Hamelin [32, p. 11] understands the causal link between two propositions as the result of applying an inference rule where the premises, the cause, constitute the "real subject" and the conclusion, the effect, is the predicate (*modus ponens* is the main rule Hamelin has in mind). Hamelin wishes to stress the point that Stoic logic is closer to Aristotle's setting as assumed by Brochard. Indeed, it is patent that Aristotle's modalities also include events (a striking witness is the famous chapter 9 of the *Peri Hermeneias*). However, Aristotle seems to think the relation from the event to the cause (if the event occurs some condition caused the event to happen), rather than the other way round: *if there is rain, there is necessarily a cause (clouds)*, but rain is not necessary! — for a lucid and thorough study on the subject see Crubellier [24]. Curiously, neither of them took into consideration ethics and Roman Law, which constituted the favorite contexts studied by the Stoics and which required an ontology of *actions* and *norms* not reducible to logical necessity.

the Aristotelian term-relation governed by the metaphysics of essences and the logic of syllogism.[5]

The propositional perspective on causal necessity allowed the Roman Jurists, and Cicero in particular, the transferring of different forms of natural causality into the realm of legal reasoning. This contributed to the inception of the notion of *ratio legis*, the cause grounding a juridical decision.[6] Perhaps one could understand the Stoic theory of signs, not as some early form of formalism but as a way to gather a general notion of cause-effect applying to both norms and events.[7]

Now two main problems arose.

1. While the predicative approach of Aristotle assured contentual relevance, the propositional construction made it difficult to tight cause and effect with bear truth-functional means. Recall the very known disputes on how to define an implication that expresses causality

2. It raised the question of the gap between norms as prescriptions (and their actualization) and propositions understood as bearers of truth; particularly so in the context of legal reasoning.

These gaps evoke the broader epistemological problem of how to link theory and experience or theory and praxis. The Arabic tradition, particularly sensitive to issues concerning *praxis*, developed the insight that the interface theory-praxis should be studied under the perspective of the dyad *prescription-actualization*, precisely in the contexts mostly cherished by the Stoics, namely ethics and Jurisprudence. The new insight of the Arabic tradition lead to the following bold steps:

- Prescriptions are understood as prescriptions to **do** rather than prescriptions that take us from one state of affairs to another: *Tun Sollen* rather than *Sein Sollen*.

- Not only events but also actions are first-class denizens of the universe of discourse. Actions and prescriptions display a contentual link that yields

[5]For a thorough discussion on Aristotle's relational view on modalities see Malink [56], who also proposes a formal reconstruction based on what he calls a *mereological pre-order semantics*.

[6]Relevant too for the development of deontic necessity within legal contexts is the concept of *conditional right* in Roman Law, one of the most important forms of legal norms in Civil Law — whereby an obligation, such as the obligation to pay some fixed amount of money, is made dependent upon some future contingent conditions (set by the benefactor in favour of a beneficiary).

[7]As mentioned above, Bréhier [16] proposes a *semiotic* reading of the logic of the Stoics that moves away from the naturalistic interpretations of Brochard [17] and Hamelin [32].

a classification of types of actions. Deontic reasoning is reasoning with content.

- Prescriptions to do are embedded in a system of hypothetical judgements involving implications where actions, the actualization of the prescriptions, are subject of predication: actions are bearers of qualifications such as law-abiding or law-breaking.[8] Similarly events are qualified as necessarily happening or possibly happening or not happening at all.

- Norms presuppose freedom of choice: A prescription to do presupposes the possibility of choosing between carrying out or not the action prescribed by the norm.

- The temporal dimension of the deontic notions condensed in the in principle *all actions are permissible unless proscribed by Law*, required the deployment of a dialectical system of argumentation called *qiyās* that regulated the integration into the legal system of an explicit updated deontic qualification (possibly different to *permissible*) for a new kind of actions.[9]

For certain, analogies between deontic, temporal and modal concepts have a long and rich history before their resurgence in contemporary deontic logic.[10] Important misses are nevertheless present in the literature on its historic sources, even in the most recent overviews, particularly so in relation to the contributions developed within Islamic jurisprudence.[11] This is still the case despite the fact that there is work on the influence of Stoicism on Arabic thinkers in general, and on the moral classification of acts as being *obligatory*, *forbidden*, *recommendable*,

[8]The notion of *conditional assertions*, provided the ground for the further sophisticated developments within the Islamic tradition of implications (including bi-implications), or *sharṭiyya muttaṣila*, and disjunctives, or *sharṭiyya munfaṣila*. For a recent thorough study of the notion of *sharṭiyya* see Hasnawi and Hodges [34, section 2.4.3, pp. 63–65].

[9]For a thorough study of theory of *qiyās* see Young [82]. The canonical form of *qiyās* is the one advocated to finding the *'illa*, or "occasioning factor" that triggered the juridical decision (such as *legally valid*) or deontic qualification (such as *forbidden or obligatory*) of a known case and transferring it to the new case. "Occasioning factor", *'illa* , is the Islamic analogue to the concept of *ratio legis* mentioned above.

[10]In fact, Knuuttila [49, p. 182] observes that Peter Abelard (1079–1144) and other early medieval philosophers often endorsed an inverted form of Leibniz's reduction by defining modal concepts by means of deontic concepts. According to this characterization, necessity is taken to be what nature demands, possibility is identified with what nature allows, and impossibility with what nature forbids.

[11]See, for example, Knuuttila [48], and the otherwise excellent essay by Hilpinen and McNamara [38, p. 14], who, though they discuss the occurrence of deontic concepts in classical Islamic jurisprudence, do not mention the early testimonies of the parallelism between deontic and modal concepts in that tradition.

reprehensible and *neutral*, including the studies by [27] and Jadaane [45].[12] In fact Gutas [30] shows that the conditions for a grounded assessment on the influence of Stoicism on Islamic thinkers, are not yet available. Indeed; Gutas [30] makes it patent that studies as the ones just mentioned are not backed by evidence stemming from the sources.

On our view, it is precisely in the context of Islamic jurisprudence that the contribution of the Arabic tradition to modality and its logic should be studied and ponderated.[13] Avicenna, who was not particularly interested in the logic of jurisprudence, seems to have influenced contemporary historic studies which focused in the developments of metaphysical rather than of deontic necessity. The time is ripe to have a closer look at legal reasoning within this tradition.

1.2 Ibn Ḥazm of Córdoba on deontic and natural necessity

Leibniz is rightly considered to be one of the most important thinkers in linking logic and legal reasoning. Especially because of his early work on a logical analysis of conditional right (1664–1669), which involves singling out one particular form of hypothetical judgement that he calls *moral implication*; and his subsequent work (in 1671) linking modal necessity with legal obligations and probability. In such a context it has been often claimed that contemporary deontic logic was born in Leibniz's *Elementa Juris Naturalis* of 1671 — see Von Wright's [79, p. 3]. Actually, it is true that Leibniz explicitly states in that work that the *transference* between deontic and modal concepts can be carried out in the following way:

Modal	**Deontic**
possible, it is intelligible.	(licitum) permissible
necessary, its negation is not intelligible.	(debitum) obligatory
possibly not, its negation is intelligible.	(indebitum) omissible
impossible, it is not intelligible.	(illicitum) forbidden

The influence of the work of Leibniz is undeniable; however the historic claim on the birth of deontic logic is inaccurate. Lameer ([50, pp. 240–241]; [51, p. 417])

[12] Jadaane [45, pp. 184–189] discusses and relativizes convincingly Van den Bergh's [75, reprinted 1987, vol. II, p. 117 of the notes] strong assertion that the *obligatory*, *recommendable*, *reprehensible* and *forbidden* notions of Islamic jurisprudence correspond (respectively) to the Stoic notions of *recte factum*, *commodum*, *incommodum*, and *peccatum*. In the same footnote Van den Bergh [75, vol. II, p. 118 of the notes] points out that Islamic theologians coupled the deontic notion of *permissible* with the modality *not logically impossible*. Van den Bergh does not develop the issue any further, however. Gutas [30] develops a thorough critical analysis of the hasty assessments by Van den Bergh and also by Jadaane.

[13] Cf. [69].

stresses this inaccuracy by pointing out that both al-Fārābī's and Ibn Ḥazm's perspectives appear to be the earliest testimony on record of a transference from deontic to modal concepts.[14]

Indeed, the passionate and staunch defence of logic within legal reasoning of the controversial thinker Ibn Ḥazm of Córdoba (384-456/994-1064) (ᶜAlī ibn Aḥmad ibn Saᶜīd ibn Ḥazm ibn Ghālib ibn Ṣāliḥ ibn Khalaf ibn Maᶜdān ibn Sufyān ibn Yazīd al-Fārisī al-Qurṭubī), had lasting consequences in the realm of legal reasoning. Moreover, his book *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples (Kitāb al-Taqrīb li-ḥadd al-manṭiq wa-l-mudkhal ilayhi bi-l-alfāẓ al-ᶜāmmiyya wa-l-amthila al-fiqhiyya)*, composed in 1025–1029, was well known and discussed during and after his time; and it paved the way for those studies that gave demonstrative reasoning a privileged place within the methods of attaining knowledge in general and legal decision-making in particular.

In fact, Ibn Ḥazm's defence of logic focused on its role for decisions in legal contexts. This lead him to advocate for a logical system that countered the "formalistic" conceptions of his time. In *al-Taqrīb* Ibn Ḥazm explicitly rejects the use of syntactic devices for the analysis of logical arguments, and attempts to develop a fully interpreted language on which arguments are built. If logic should have any role in real legal practice, then it should be built on the study of paradigmatic real cases of juridical decisions. In order to do so, he undertakes a thorough study of deontic notions and their modal counterparts, that makes him one of the fathers of the logic of norms.

The following extract from Ibn Ḥazm, *al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-Alfāẓ al-ᶜĀmmiyya wa-l-Amthila al-Fiqhiyya,* ed. Aḥmad b. Farīd b. Aḥmad al-Mazīdī, (Beirut: Manshūrāt Muḥammad ᶜAlī Bayḍūn, Dār al-Kutub al-ᶜIlmiyya, 2003), pp. 83-84, constitutes the main historical source of the parallelism within the Arabic tradition.

[14]Lameer [51, p. 306] acknowledges Gutas [30] for the reference to Ibn Ḥazm

Translated by Walter Edward Young.[15]

Chapter on Elements (*ᶜanāṣir*)	باب العناصر
Know that the elements (*ᶜanāṣir*) of all things (*ashyā'*)—that is, their classes with regard to making assertions (*ikhbār*) about them—are of three classes, there being no fourth.	اعلم ان عناصر الأشياء كلها أي أقسامها في الاخبار عنها ثلاثة أقسام لا رابع لها
[They are] either necessary (*wājib*), being such as are necessary and manifest, or from among such as must be, like the rising of the sun each morning, and the like of that, such being called in God's laws 'obligatory' (*farḍ*) and 'binding' (*lāzim*);	إمّا واجب وهو الذي قد وجب وظهر أو ما يكون مما لا بد من كونه كطلوع الشمس كلَّ صباح وما أشبه ذلك وهذا يسمى في الشرائع الفرض واللازم
or possible (*mumkin*), being such as might be and might not be, like our anticipation that it will rain tomorrow, and the like of that, such being called in God's law 'lawful' (*ḥalāl*) and 'permitted' (*mubāḥ*);	وإما ممتنع وهو الذي لا سبيل إليه كبقاء الإنسان تحت الماء يوماً كاملاً أو عيشه شهراً بلا أكل أو مشيه في الهواء بلا
or impossible (*mumtaniᶜ*) being such as to which there is no path, like a human's remaining under water for an entire day, or his living a month without food, or his walking in the air without some cunning artifice, and the like of that. And this is the type of thing that, if we saw it manifest in a human, we would know he is a prophet; and this class is called in God's laws 'forbidden' (*ḥarām*) and 'prohibited' (*maḥẓūr*).	وإما ممتنع وهو الذي لا سبيل إليه كبقاء الإنسان تحت الماء يوماً كاملاً أو عيشه شهراً بلا أكل أو مشيه في الهواء بلا حيلة وما أشبه ذلك وهذه التي إذا ظهرت من إنسان علمنا أنه نبي وهذا القسم يسمى في الشرائع الحرام والمحظور
Furthermore, the possible (*mumkin*) is divided into three classes, there being no fourth:	ثم الممكن ينقسم اقساماً ثلاثةً لا رابع لها

[15]From, [69].

the nearly possible (*mumkin qarīb*), like the possibility of occurrence of rain upon a condensing of clouds in the two months of *Kānūn* (I.e., December and January.), or the victory of a large number of the courageous over a small number of the cowardly;	ممكن قريب كإمكان وقوع المطر عند تكاثف الغيم في شهري كانون وغلبة العدد الكبير من الشجعان العددَ اليسيرَ من الجبناء
and the distantly possible (*mumkin baᶜīd*), which is like the defeat of a large number of the courageous at the hands of a small number of the cowardly, and like a cupper (*ḥajjām*) [i.e., a practitioner of cupping] taking charge of the Caliphate, and the like of that;	وممكن بعيد وهو كانهزام العدد الكثير من الشجعان عند عدد يسير من جبناء وكحجام يلي الخلافة وما أشبه ذلك
and the purely possible (*mumkin maḥḍ*), whose two extremes are equal, such being like one standing—either he will walk or he will sit—and the like of that.	وممكن محض وهو يستوي طرفاه وهو كالمرء الواقف إما يمشي وإما يقعد وما أشبه ذلك
And likewise we find that this middle class [i.e., the *mumkin*, corresponding to the *mubāḥ*] is, in God's laws, divided into three classes: recommended-permitted (*mubāḥ mustaḥabb*); reprehended-permitted (*mubāḥ makrūh*); and evenly permitted (*mubāḥ mustawin*) having no tendency towards one of the two sides.	وكذلك نجد هذا القسم المتوسط في الشرائع ينقسم أقساما ثلاثة فمباح مستحب ومباح مكروه ومباح مستوٍ لا ميل له إلى أحد الجهتين
As for recommended-permitted (*mubāḥ mustaḥabb*), it is such that when you do it you are rewarded (*ujirta*), but if you neglect it you do not sin (*lam ta'tham*) and you are not rewarded; like praying two supererogatory prayer-cycles, voluntarily.	فأما المباح المستحب فهو الذي إذا فعلته أجرت واذا تركته لم تأثم ولم تؤجر مثل صلاة ركعتين نافلة تطوعاً

And as for reprehended-permitted (*mubāḥ makrūh*), it is such that when you do it you do not sin and you are not rewarded, but if you neglect it you are rewarded; and that is like eating while reclining, and the like.	وأما المباح المكروه فهو الذي إذا فعلته لم تأثم ولم تؤجر وإذا تركته أجرت وذلك مثل الأكل متكئاً ونحوه
And as for evenly permitted (*al-mubāḥ al-mustawī*), it is such that when you do it or you neglect it you do not sin and you are not rewarded; and that is like dyeing your garment whichever colour you please, and like your riding whichever beast of burden you wish, and the like.	وأما المباح المستوي فهو الذي [إذا] فعلته أو تركته لم تأثم ولم تؤجر وذلك مثل صبغك ثوبك أي لون شئت وكركوبك أي حمولة شئت ونحوه

The basic units of Islamic deontic logic are what we might call, indulging in terminological anachronism, *heteronomous imperatives*. The point of *heteronomous imperatives*, is to develop a logic of norms where the contentual analysis of deontic qualifications such as *Obligatory*, *Forbidden*, *Permissible*, *Facultative*, are put into practice, in order to justify transferring a juridical decision from a known case to an unknown one. Islamic deontic notions qualify the performance of actions as worthy of being *rewarded (in different degrees), sanctioned or neither* of them. In a more modern and general framework we might use the qualifications *law-abiding*, *law-breaking* and legally neutral (neither *law-abiding* nor *law-breaking*) instead[16] — on the grounds of which the agent might be sanctioned or not.[17] Other possibilities are sanctioned by law, not-sanctioned by law and legally neutral, or within a value-approach to Law; *legally worthy*, *legally unworthy*, *legally worth-neutral*.[18]

We claim that the logic of heteronomous imperatives that grows out from Ibn Ḥazm's insight, when shaped by Martin-Löfs Constructive Type Theory and some suitable generalization of the deontic qualifications, suggests a way out of many of the paradoxes of deontic logic. Moreover, if we combine the logic of heteronomous imperatives with the dialectical system of parallel reasoning, known as, *quiyās*, developed by the early teachers of Ibn Ḥazm,[19] the path is open to launch a new approach to legal reasoning informed by the science of Law with a *special emphasis on factors in play when it falls into error, when bad reasoning is mistaken for good* — John Woods [80, p. 1]. Apparently, the proposal goes far beyond the devices and scope of Ibn Ḥazm and of the Islamic Jurisconsults of the

[16]The idea of substituting *reward* and *sanction* by *law-abiding* and *law-breaking* was suggested to Rahman by Zoe McConaughey.

[17]Notice that according to this interpretation, though a performance can be neither law-abiding nor law-breaking, the agent of the performance, will be sanctioned iff his/her performance breaks the law.

[18]Value-systems within legal reasoning are often seen as competing with logical ones.

[19]In fact, in his main work Ibn Ḥazm rejected *quiyās* developed by his first teachers, for being too prone to falling into logical pitfalls and arbitrariness — cf. Ibn Ḥazm [42, pp. 144; 160–166], [21, pp. 67–68].

Middle-Ages. Still, much can be learned from the lessons our Elders cared to bequeath us.

The following is an invitation to explore together this new path. It does not provide a totally worked out theoretical framework. Much is still to be done. Nevertheless, we hope that what we brought forward will induce some reader to take up the gauntlet.

2 Ibn Ḥazm's Heteronomous Imperatives

> *There is in the orderly procedures of the law a good deal for the traditional logician and epistemologist to mull over and quite possibly to learn from.*
>
> *John Woods, [80, p. 5]*

2.1 The main definitions

Muslim jurists identified five deontic qualifications for an action. Ibn Ḥazm defines them as follows:[20]

1. **wājib, farḍ, lāzim.** Obligatory action is the one which:
   - If we do it we are rewarded.
   - If we do not do it we are sanctioned.
2. **ḥarām, maḥẓūr.** Forbidden action is the one which:
   - If we do it we are sanctioned.
   - If we do not do it we are rewarded.
3. **mubāḥ mustaḥabb.** Recommended permissible action is the one which:
   - If we do it we are rewarded.
   - If we do not do it we are neither sanctioned nor rewarded.
4. **mubāḥ makrūh.** Reprehended permissible action is the one which:
   - If we do not do it we are rewarded.
   - If we do it we are neither sanctioned nor rewarded.
5. **mubāḥ mustawin.** Evenly permissible action is the one which:
   - If we do it we are neither sanctioned nor rewarded.
   - If we do not do it we are neither sanctioned nor rewarded.

[20] Ibn Ḥazm (1926-1930, vol. 3, p. 77); [42, p. 86]; [44, pp. 83–4].

Note that the classification assumes that reward and sanction are incompatible but not contradictory. Some actions can be neither rewarded nor sanctioned; and this latter point is crucial for the introduction of values and degrees.

Notice too that whereas the notion of *sanction* corresponds to the vocabulary of contemporary European jurisprudence, the notion of *reward* at work in the classification of actions seems to have its origins in the realm of theology.[21] The point is simply that, as thoroughly developed by Hallaq [31], the inseparable "groundwork" for the emergence of both Islamic morality and law is in the Qurʾān. Nevertheless a not theological interpretation of *reward* in some legal contexts is possible, such as in the case of conditional right, where a beneficiary can be said to be "rewarded" with a good, if some condition, specified by the benefactor has been satisfied.

Actually, Ibn Ḥazm's extension of *mubāḥ*-permissibility into the categories of recommended and reprehended is atypical. All forms of "permissibility" have a value; that is, in terms of doing the recommended or not doing the reprehensible, both surpass the neutral value of the "evenly permitted," while not yet reaching the value of doing the obligatory and not doing the forbidden. At the same time, neither doing the reprehensible nor neglecting the recommended descends below the neutral value of the "evenly permitted," which latter, always above the status of doing the forbidden and neglecting the obligatory, remains steadfastly in the middle.

Interestingly, Ibn Ḥazm's classification of actions varies in relation to others, such as that of the prominent Muᶜtazilite, the Qāḍī ᶜAbd al-Jabbār (324-415/935-1025), in his *Mughnī* (vol 11–14) and in his *al-Uṣūl al-Khamsa*, pp. 79-96).[22]

- *A* is evil (***qabīḥ***) if and only if the doer deserves blame.
- An act *A* is an act of grace (***tafaḍḍul***) or recommended (***nadb***) if and only if the doer deserves praise, and the omitter does not deserve blame.
- *A* is merely permissible [or optional] (***mubāḥ***) if and only if neither the doer nor the omitter deserves blame or praise.
- *A* is obligatory (***wājib***) if and only if the omitter deserves blame.

The last three kinds of actions are described as "good" (*ḥaṣan*) actions, and the set seems to be lacking the category of *reprehended*. However, ᶜAbd al-Jabbār, instead of distinguishing a special category for the *reprehended*, introduces the category of *not obligatory (ghayr wājib)*, characterizing all those actions for which the omitter is not blamed — including the *evil*, *the permissible* and *the recommended*.

It is worth mentioning that Hilipinen and McNamara [38, p. 1], who briefly discuss this classification of ᶜAbd al-Jabbār, point out that it is very close to Alexis Meinong's logic of norms, with the exception of a missing category of excusable actions—such being precisely the category which is included in Ibn Ḥazm's classification mentioned above!

[21] Cf. [35, pp. 74–75].

[22] We owe the citation to Hourani [40, pp. 99–102], who extracted these definitions from the cited texts.

2.2 Freedom and Heteronomy: Ought presupposes Can

The following approach is based on the insight that the most salient characteristics of deontic imperatives listed above are:

> **Assumption of freedom of choice, or takhyīr:** the fact that an action can be chosen to be performed or not.
>
> **The heteronomy of imperatives:** the fact that the way actions are qualified by reward or sanction depends upon the choices made.

Both conditions are linked to the idea of responsibility that is at the core of Ibn Ḥazm's understanding of obligation. This point has been stressed by Hourani [40, p. 175] as follows:

> *The fact that concerns us in a historical account is that in all ethical contexts [Ibn Ḥazm] regards man as responsible for his own actions and liable to Reward and Punishment accordingly.*

In our understanding, responsibility manifests itself in the fact that a legally accountable individual can not only choose to do or not to do some kind of action, but he can also choose not to choose at all; the actions must be contingent on us: we need not necessarily accept the choice. On the other hand, reward and sanction are both dependent on the choices made.

In fact, Islamic Jurisprudence makes explicit the presuppositions for the application of a deontic qualification. Indeed, classifications such as obligatory, forbidden, and permissible, grounding a juridical decision (ḥukm) for a particular action (e.g., it is forbidden to eat pork), presuppose that:

- (a) the person who performs an action is legally accountable (*mukallaf*);
- (b) the action in question is one for which the liberty to choose between carrying it out or not has been given (*takhyīr*).

Notice that this approach is quite different from current studies in deontic logic that include, as axiom, the implication $\mathbf{O}A \supset \mathbf{M}A$ — where "**O**" stands for "obligatory" and "**M**" for "possible," known as the principle that *Ought implies Can*, and also dubbed *Kant's principle (Sollen-Können-Prinzip)*.[23] According to our analysis of the Islamic conception, however, we find that:

- Every deontic qualification, and not only the obligatory, **presupposes rather than implies** that the qualified action is allowed to be chosen.[24]

[23] Cf. Prior [66], von Wright [78, pp. 108–16; 122–25], Hilpinen [36, pp. 14–15], , Chellas [22], al-Hibri [7, pp. 18–21], Hilpinen and McNamara [38, p. 38].

[24] Notice that Hintikka's [39, p. 86] analysis of Kant's principle is quite close to our view of the role of *takhyīr* — though he speaks of **non logical consequence** rather than presupposition:

> *Our result is in itself very simple, and may even appear trivial — after it has been*

So, *ought presupposes can*. However; if *can* is understood as some general form of *Permissibility*, then all actions qualified as *mandatory* are also permissible. In such contexts permissible is defined as including all those actions that qualify for reward (i.e. those that are rewarded when carried out and all those other ones that are rewarded when not carried out).[25] Still, there is another sense of "**can**" involved in Kant's principle, namely, as *ability to fulfil the duty*, that triggers some known puzzles of current deontic logic.[26] We will briefly come back to this issue in the last sections of our paper.

The logical upshot of all this is that the underlying structure is that of a hypothetical, such that if we accept to make the choice between performing or not performing a certain action, we are rewarded or sanctioned in relation to this choice.

In this context, let us recall that in the Arabic tradition propositional logic involves the study of *sharṭiyya* propositions, usually translated as conditionals. The compounds of judgements involving a *sharṭiyya* proposition are not asserted, but simply "connected". This differentiates judgements involving sharṭiyya propositions from those involving attributives, called *ḥamliyya* propositions, whereby a predicate is asserted for a subject. *Conditionals* are subdivided into that type constituted by implication (or bi-implication), called *sharṭiyya muttaṣila*, and that type which is constituted by disjunctives (exclusive or inclusive), called *sharṭiyya munfaṣila*.[27] If we take the stance that *connecting without asserting* amounts to *making the truth of the consequent dependent upon the truth of the antecedent*, we might formulate the subdivision as follows:

- The truth of the consequent of a conditional judgement constituting the implication C provided A, is dependent upon the truth of the antecedent, which is not (yet) determined to be true.
- The truth of the consequent of a conditional judgement involving a disjunction as

established. (It ought to be the case that all duties are fulfilled. Hence it ought to be possible to fulfil them). Some additional interest is in any case lent to our observations by the possibility that the 'sollen-können' principle was perhaps right from the beginning intended, however dimly and inarticulately, as an expression of a deontic consequence rather than a logical consequence. The principle was brought to prominence in moral philosophy by Kant. Hence we have to ask: how did he conceive of it? Kant's explanations are not distinguished by their lucidity, but an unmistakable and recurrent turn of thought in Kant is in any case a connection between the 'ought implies can' principle and the concept of freedom. (See e.g. Critique of Pure Reason A 807, Critique of Practical Reason, 1st ed., p. 54.) Moral freedom, for Kant, lies in the very fact that a man can act in the way he ought to act.

[25] In fact, in his al-Iḥkām fī uṣūl al-Aḥkām (vol. 8, p. 101), Ibn Ḥazm seems to extend his deontic system with notions of *forbidden to do* and *obligatory not to do*, based only on what is *permissible to do* or *not to do*.

Forbidden is all that is not permissible to do,
obligatory is all that is not permissible not to do.

[26] Cf. [38, pp. 67–69].

[27] Cf. Rescher [72, pp. 76–78], and Jadaane [45, pp. 117–21]. For a recent, thorough study of the notion of *sharṭiyya* see Hasnawi and Hodges [34, Section 2.4.3, pp. 63–65].

premise is dependent upon the truth of one of its sides, which is not (yet) determined to be true.

This already suggests the main idea behind our analysis of judgements involving deontic and modal concepts. In a nutshell, our point is to analyse such judgements as a conjunction of two implications, such that the truth of the antecedent of each of these implications is dependent upon a disjunction. Take the case of the conditional expressing an obligation. This *conditional* is constituted by the following implications:

- If an action x of type A is performed, then it will be rewarded; and if it is omitted, then it will be sanctioned (omitting to perform A has been established by the legal system as triggering a sanction, i.e. the contrary of reward) — under the hypothesis that an action of type A can be performed or omitted.

Similarly, for the case of the necessity as applied to events — assuming that cause is both necessary and sufficient):

- If event E occurs, then it satisfies some condition(s) C that causes the event to happen. If the event is absent, then C will be absent too (the absence of C has been established as inhibiting the occurrence of events of the type E) — provided that both, event and causal conditions can or cannot take place.

- If event E occurs, there is some condition C that causes the event to happen or not to happen. However the condition tends to occasion E rather than not (there are more cases verifying the occurrence of E than its absence). If the causal condition is absent, then E will be absent too.

- If event $\neg E$ is the case, there is some causal condition C that occasions the event to happen or not to happen. Event E is absent, despite the fact that the condition tends to occasion E rather than not. If the causal condition is absent, then E will be absent too.

- If event E occurs, there is some causal condition C that occasions the event to happen or not to happen. There is no tendency of C to occasion one or the other alternative.

The parallelism between deontic and metaphysical modalities develops naturally from the idea of comparing the *degree of likeness* of an event to happen with the *degree of deontic enforcement* involved by a command. The source of the notion of likeness to happen seems to be Aristotle's [3]), *Peri Hermeneias*, chapter 9, 19a18-22) distinction between different cases of contingent events, some of them for *the most part and commonly, tend in a certain direction, and yet they may issue at times in the other rarer direction.* Moreover, Ammonius famous commentary of this passage of Aristotle is strikingly close to the passage by Ibn Ḥazm on the possible, quoted above:[28]

[28]We owe this reference to Carlo Natali (Venezia), who pointed out this passage of Ammonius

> *The contingent is divided into three: one is called 'for the most part' (hôs epi to polu), for example that a man is born with five fingers or becomes gray with age (for things behaving otherwise are rare); another is 'for the lesser part' (hôs ep' elatton), for example that one digging comes upon a hoard; and the last is 'equally <often>' (ep' isês), for example to bathe or not to bathe and to walk or not to walk.* Ammonius [8, 142.1]).

We cannot discuss in the present paper the parallelism any further — for the development of the parallelism see Rahman, Farid and Young [69]. In the following sections we will instead work out the logical analysis of the deontic modalities based on Per Martin-Löf's [59] study of judgements. In accordance with Martin-Löf's terminology, we will not here employ the term *conditional* for judgements involving implications or disjunctions, but, rather, *hypothetical*, which stresses the point that the compounds of such judgements are not yet known to be ***true***. Thus, according to this terminology, *sharṭiyya* denominates general *hypothetical* judgements, which can be constituted by implications, or bi-implications (*sharṭiyya muttaṣila*), and/or disjunctives (*sharṭiyya munfaṣila*). Let us not now furnish the main formal elements of Martin-Löf's theory underlying our interpretation.

2.3 Deontic Imperatives and the CTT-Analysis of Hypotheticals

Per Martin-Löf's [59] Constructive Type Theory (CTT) provides a thorough formal framework whereby categorical and hypothetical judgements can be explicitly distinguished at the object-language level without conflating judgements with the propositions that constitute them.

Since these distinctions are crucial for the formal reconstruction of traditional logic in general—and of the Arabic tradition in particular—we have chosen to employ the language of CTT for our logical study on the origins of deontic concepts. More precisely, the CTT-framework allows one to distinguish, at the language level, both the *taṣawwur* of a judgement, i.e., its conceptualization or (roughly) proposition, and its *taṣdīq*, or assent, i.e., the act of judgement itself, or, sometimes, the linguistic expression of that act.[29]

Let us first briefly introduce the formal instruments we will make use of.

2.3.1 Categorical and Hypothetical Judgements in CTT

On Categorical Judgements.

In the CTT framework it is possible to express at the object-language level

$$A \textbf{ true},$$

which, when asserted by some individual **g**, conveys the information that this individual is in possession of some proof-object for A. Moreover, it can be rendered explicit by means of the **categorical judgement**

to Rahman in a personal email as a follow up of a presentation at the Universidad Panamericana, Mexico-City, September 2018.

[29] Cf. [34, pp. 56–57].

$d : A$,

which reads, d is a proof (object) d of A — or the individual **g** can bring forward the proof-object d in support of his claim that A is ***true***.[30]

More generally, within CTT a proposition is interpreted as a set, the elements of which represent the proofs of the proposition, the solution to a problem, the fulfilments of an expectation.[31] Accordingly,

$$d : A \qquad\qquad A \textbf{ true}$$

can be read as

d is an element of the set A	A has an element
d is a proof of the proposition A	A is **true**
d is a solution to the problem A	A has a solution
d fulfils the expectation A	A is fulfilled

Ranta [71, p. 54] combines CTT with Davidson's [25, essays 6–10] idea that an individual action makes an action-proposition true. Accordingly the proposition

(that) Al-Fārābī read Aristotle's *Analytica Posteriora*

is made ***true*** by individual readings of Al-Fārābī performing actions of that type. This interpretation is not far from the interpretation mentioned above of expectations as propositions and fulfilments as proof-objects.

- We will follow here Ranta's suggestion and assume that we have action-propositions that are made ***true*** by some evidence that some action of the type expressed by those propositions has been performed.[32]
- Notice that this not only fits nicely with Ibn Ḥazm's original text (see appendix), where he uses the term *al-ashyā'*, "things," to include actions and events, but, as mentioned in the preface and discussed below, it is a consequence of the insight that deontic and modal concepts qualify both actions and events.

[30] See Martin-Löf [59, pp. 9–10]. For a short introductory survey see [70, Chapter II].

[31] This array of readings is due to combining the Curry-Howard correspondence between propositions and sets with Heyting's proof-theoretical interpretation of propositions. So within CTT a proposition is interpreted as a set whose elements represent the proofs of that proposition. It is also possible to view a set as a problem description in a way similar to Kolmogorov's explanation of the intuitionistic propositional calculus. In particular, a set can be seen as a specification of a programming problem, the elements of the set are then the programs that satisfy the specification — see Martin-Löf [59, p. 7]. Furthermore in CTT sets are understood also as types so that propositions can be seen as data- (or proof-) types.

[32] Strictly speaking, it is the performance itself, rather than the evidence for the performance that is subject of deontic qualifications. However, on one hand, within legal contexts, it is the evidence for the carrying out of an action, that grounds sanction or exoneration of sanction, on the other actual performances of actions seem to be the analogue of *executions of programs* in CTT — see Martin-Löf [61] — though executions have priority in relation to meaning, reasons or evidences take the lead when actions turn into action-propositions.

On Hypothetical Judgements.

One of the characteristic features of CTT is that it also allows, at the object-language level, expression of **hypothetical judgements** as a form of statement distinguishable from the assertion of the truth of an implicational proposition. Hypothetical judgements give rise to dependency structures in CTT, such as

$$B(x)\,\mathbf{true}\,(x : A)$$

or, in its explicit form:

$$b(x) : B(x)\,(x : A),$$

which reads: $b(x)$ is a (dependent) proof object of $B(x)$, provided x is a proof object of the proposition A.

Or: the function b takes elements from the set A, and yields proof-objects for $B(x)$.[33]

In other words, in this frame, the dependence of the truth of B upon the truth of A amounts to the dependence of the proof-object of B upon the proof-object of A. And the dependence of the proof object of B upon the proof-object of A is expressed by means of the function $b(x)$ (from A to B), where x is a proof-object of A and where the function $b(x)$ itself constitutes the dependent proof-object of B.

In our context, we have the set of (evidences of) performances of actions A, and the set R of rewards (or reward-actions). Thus, the expression

$$b(x) : R(x)\,(x : A),$$

can be read as:

> The function b provides evidence for a proposition of the form (*the performance*) x will be rewarded

Thus, if we have $b(x) : R(x)\,(x : A)$ as a premise, and we have as a second premise the fact that indeed that there is a performance a of the action-proposition A (i.e., if we have as premise $a : A$), then we can infer that performance a will be rewarded (i.e., $b(a) : R(a)$).

In plain words, from the premises

> some performance x of an action will be rewarded, provided it is the performance of an action of the type A;

and

> a is such a performance $(a : A)$;

we can infer:

> performance a is rewarded $(b(a) : R(a))$.

[33] For example, intuitively, if A is the set of natural numbers and B is the set of whole numbers, then the function takes one natural number and yields an element of the set of whole numbers B, e.g. $b(x) = 2x$.

$$\frac{a : A \quad b(x) : R(x)\,(x : A)}{b(a) : R(a)}$$

Similarly for sanctions

$$\frac{u : \neg A \quad c(z) : S(z)\,(z : \neg A)}{c(u) : S(u)}$$

Moreover, the existence of b and c mean precisely that A is obligatory. What is still lacking is the idea that the reward is made dependent on the occurrence of a *future contingent action*. In order to implement this task, we will supply the antecedent of the hypothetical with a richer structure than the one discussed above. More specifically, we take it that the antecedents of hypotheticals underlying deontic imperatives have the form of a constructivist disjunction. That is, a disjunction such as whose proof-object amounts to indicating explicitly which of either disjunct obtains. Thus the hypothesis looks like:

$$x : A \vee \neg A$$

(where x stands for some piece of evidence for either carrying out an action of type A, or for omitting to perform it).

Since we are in the context of a constructive disjunction, its truth requires that we know which of either disjunct obtains. Different to classical logic, the disjunction $A \vee \neg A$ is not per se assumed to be ***true***. This disjunction is to be conceived as a presupposition of the distribution of reward and sanction on actions set by the legal system. So in such a formal system, facing the choice of performing or not performing a given type of action can be rendered explicit in a quite straightforward manner.

In the context of our reconstruction, omitting to perform an action that instantiates the action-proposition A (i.e., not doing it) is conceived of as frustrating the performance of an action of that type, e.g., *stopping (or inhibiting) eating or drinking when a day of fasting in Ramadan begins.* This interpretation is close to the notion of aborting a process found in the constructivist understanding of negation (see Martin-Löf 1984, p. 36).

What we now need is to express the dependence of the rewarding or sanctioning upon the choice made. More precisely, if we are describing an *obligatory action*, what we need to express is the following:

Obligatory action

> *If there is some evidence that the individual* **g** *made the choice to perform an action of type A (i.e., if there is evidence that he made the choice for the* ***left side*** *of the disjunction) then he is rewarded (for this performance).*
>
> *If there is some evidence that the individual* **g** *made the choice to omit performing an action of type A (i.e., if there is evidence that he made the choice for the* ***right side*** *of the disjunction) then he is sanctioned (for this omission).*

If we pull all this together and deploying the abbreviation $=_{\{H\}}$ for $A \vee \neg A$ we obtain:

$b(x) : [\,(\forall y : A)\,\mathbf{left}^{\vee}(y) =_{\{H\}} x \supset R(y)\,] \wedge [\,(\forall z : \neg A)\,\mathbf{right}^{\vee}(z) =_{\{H\}} x \supset S(z)\,]\,(x : A \vee \neg A)$

Where:

> the expressions "$\mathbf{left}^{\vee}(y)$" and "$\mathbf{right}^{\vee}(z)$" stand for the injections that render the disjunction $A \vee \neg A$ true.[34] Whereas "$\mathbf{left}^{\vee}(y) =_{\{H\}} x$"stands for the choice of performing an action of the type (of the action-proposition) A, and "$\mathbf{right}^{\vee}(z) =_{\{H\}} x$" stands for the choice of not performing such a type of action-proposition.[35]

The identity expression can be glossed as follows:

> *The piece of evidence that renders true the left (right) of the disjunction is identical to the evidence for the carrying out an action of the type (of the action-proposition) A (for not carrying out A).*

Thus, if we add the identity condition to our gloss, the expression

$b(x) : [\,(\forall y : A)\,\mathbf{left}^{\vee}(y) =_{\{H\}} x \supset R(y)\,] \wedge [\,(\forall z : \neg A)\,\mathbf{right}^{\vee}(z) =_{\{H\}} x \supset S(z)\,]\,(x : A \vee \neg A)$

reads:

> All those performances of an action of type A identical to the ones chosen (by agent **g**) to be performed (i.e., if the left side of the disjunction has been chosen to be performed), are to be **rewarded**.
> All those cases omitting to perform an action of type A identical to the ones chosen (by agent **g**) to be omitted (i.e., if the right side of the disjunction $\neg A$ has been chosen to be performed), are to be **sanctioned**.

However, in the context of Islamic Law, omitting to perform an obligatory action, or performing a forbidden one, is sanctioned **if the omission *z* is not excused**; i.e., provided that$\neg E(z)$ applies (the prescribed fasting during Ramadan is not obligatory, for example, while travelling) — such a proviso is also very important in contemporary European Civil Law. The proviso can be integrated into the hypothetical as follows:

$(\forall y : A)\,[\,\mathbf{left}^{\vee}(y) =_{\{H\}} x \supset R(y)\,] \wedge [\,(\forall z : \neg A)\,(\neg E(z) \wedge \mathbf{right}^{\vee}(z) =_{\{H\}} x \supset S(z))\,]\,\mathbf{true}\,(x : A \vee \neg A).$

Actually, in the present paper we leave $E(x)$ out, since this relates to defeasibility, an issue that is linked to the dynamics of argumentation, which we deal elsewhere — Rahman and Iqbal [67] discuss defeasibility in the context of legal argumentation in Islamic Law, though they do not combine it with an analysis of the deontic qualifications of actions.

[34] We have slightly changed the notation for injections, which when they occur as proof-objects of a disjunction usually take the notation $i(x)$ and $j(x)$ — see Ranta [71, p. 47].

[35] Cf. [71, p. 52–53].

2.3.2 Ibn Ḥazm's Heteronomous Imperatives

As already mentioned, deontic qualifications of actions presuppose that the performer is legally accountable and the performer has been given the liberty to choose (*takhyīr*) between two alternatives. The CTT-framework for hypotheticals provides the formal means to express

> that the deontic qualifications assume such a choice,
> that the definition does not assume that such a choice has been made.

Notice that the notion of "allowance to choose" involved in *takhyīr* is different from the notion of *permissible*, which latter applies to an action already chosen. Permissibility, as with all other deontic qualifications, presupposes the *liberty to choose*.

In other words, the choice alternatives constitute the assumption of a hypothetical. Strictly speaking, each of the deontic concepts determines a **subset** of a general set of actions in a straightforward manner. For instance,

- **Obligatory** is the set of all those actions rewarded when performed and sanctioned when omitted.

If we elaborate this for all deontic qualifications we obtain:

wājib, farḍ, lāzim:
If we do it we are rewarded.
If we do not do it we are sanctioned
$b_1(x) : [\,(\forall y : A_1)\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset R_1(y)\,] \wedge [\,(\forall z : \neg A_1)\,\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset S_1(z)\,]\,(x : A_1 \vee \neg A_1)$

ḥarām, maḥẓūr:
If we do it we are sanctioned.
If we do not do it we are rewarded.
$b_2(x) : [\,(\forall y : A_2)\,\mathbf{left}^{\vee}(y) =_{\{H2\}} x \supset S_2(y)\,] \wedge [\,(\forall z : \neg A_2)\,\mathbf{right}^{\vee}(z) =_{\{H2\}} x \supset R_2(z)\,]\,(x : A_2 \vee \neg A_2)$

mubāḥ mustaḥabb:
If we do it we are rewarded.
If we do not do it we are neither sanctioned nor rewarded.
$b_3(x) : [\,(\forall y : A_3)\,\mathbf{left}^{\vee}(y) =_{\{H3\}} x \supset R_3(y)\,] \wedge [\,(\forall z : \neg A_3)\,\mathbf{right}^{\vee}(z) =_{\{H3\}} x \supset (\neg S_3(z) \wedge \neg R_3(z))\,]\,(x : A_3 \vee \neg A_3)$

mubāḥ makrūh:
If we do it we are neither sanctioned nor rewarded.
If we do not do it we are rewarded.
$b_4(x) : [\,(\forall y : A_4)\,\mathbf{left}^{\vee}(y) =_{\{H4\}} x \supset (\neg S_4(y) \wedge \neg R_4(y))\,] \wedge$
$[\,(\forall z : \neg A_4)\,\mathbf{right}^{\vee}(z) =_{\{H4\}} x \supset R_4(z)\,]\,(x : A_4 \vee \neg A_4)$

mubāḥ mustawin:
If we do it we are neither sanctioned nor rewarded.
If we do not do it we are neither sanctioned nor rewarded.

$b_5(x) : [\,(\forall y : A_5)\,\mathbf{left}^{\vee}(y) =_{\{H5\}} x \supset (\neg S_5(y) \wedge \neg R_5(y))\,] \wedge [\,(\forall z : \neg A_5)$
$\mathbf{right}^{\vee}(z) =_{\{H5\}} x \supset (\neg S_5(y) \wedge \neg R_5(y))\,]\,(x : A_5 \vee \neg A_5)$

In some contexts, it might be desirable to define deontic qualifications as expressions building propositions. In fact, it is quite straightforward, since a hypothetical is one inference away from a universal:

$(\forall x : A_1 \vee \neg A_1)\,\{\,[\,(\forall y : A_1)\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset \neg R_1(y)\,] \wedge [\,(\forall z : \neg A_1)$
$\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset S_1(z)\,]\,\mathbf{true}$

Similar applies to Forbidden and Permissible.

Thus, the whole expression can form new propositions in the usual way, for example as consequent of some implication, and so on.

2.4 Ought to do rather than Ought to be and the puzzles of Deontic Logic

> *The law is not a theoretical abstraction. It is a concrete reality in the everyday management of a country's complex polity. We should pay attention to this.*
>
> *John Woods [80, p. 1–2]*

As mentioned, we share von Wright's [79, p. 34] qualification of traditional logical analysis of norms concerning

> [...]*structures resembling what Kant called hypothetical imperatives*

though we certainly take exception to the remark that such approaches do no constitute a *genuine deontic logic* — if the remark cannot be reduced to the obvious assertion that the traditional logic of norms is different from the analysis delivered by contemporary formal semantics.

Our incipient exploration, based on Ibn Ḥazm's analysis of legal norms, does not yet deliver a logic of norms, but a logical analysis of deontic notions, where obligation can be defined both as a particular kind of inference, namely a hypothetical judgement, and as an operator, namely as a universally quantified expression (a $\prod$-type).[36] The point of the logical analysis of deontic notions in such context is shaping the argumentation in favour or against transferring the juridical decision in relation to a known case to a new one. So, according to our approach, the main focus of deontic qualifications is contributing

[36] As pointed out by Hilpinen and McNamara [38, pp. 25–31], contemporary Deontic logicians have often made a distinction between two interpretations of deontic sentences. It has been suggested that a deontic sentence p can be interpreted normatively (or prescriptively) as expressing a mandatory norm, or descriptively as a statement that it is obligatory that p, according to some unspecified system of norms or law (see [78, viii, pp. 104–5]; Stenius [74, pp. 250–1]; Alchourrón [5, pp. 243–5]; Alchourrón and Bulygin [6, p. 121]. It might be argued that our framework is closer to the descriptive interpretation — though perhaps our distinction between the type of action and its performance might offer some middle-way.

to both the content of legal norms and their implementation, rather than studying the logical validity of arguments involving deontic qualifications.

To formulate the point a bit differently, our logical analysis of the deontic content of legal norms takes the side of those how prioritize **ought to do** (*Tunsollen*) over the Leibnizian concept of **ought to be** (*Seinsollen*).[37] While it seems to be natural to endorse the assertion that *it ought to be the case that contradictions are false*, it is not at all clear how this notion of obligation is linked to *what agents ought to do or bring about.*

A direct consequence of this perspective is that in such a context it does not makes sense to include tautologies within the set of obligatory legal norms,[38] a standard problematic issue in standard deontic logic. Similarly, some other some known paradoxes of current standard deontic logic do not seem to arise. Let us start by studying the case of tautologies before engaging in other puzzles of classical deontic logic.

2.5 On legal content and the legal irrelevance of logical relevance

2.5.1 Neither tautologies nor logical truths are legally obligatory statements

Woods [80, pp. 177–188] stresses the fact that there seems not to be a lot for a lawyer to learn about relevance in relevant logics of the Anderson and Belnap sort, despite the fact that relevance, of the legal kind, has a crucial role to play in legal trials. According to Woods [80, p. 182] the kind of relevance involved in legal trials is about *worth-hearing*, that is, if the piece of evidence at stake, contributes to the decision-making procedure. Woods' notion of *legal relevance*, is content driven (he speaks of *materiality*) rather than logical or reducible to Bayesian probability.

Now, Woods' [80, Chapters XIV, XV] focus is the relevance of a piece of evidence during a trial, whereas our central aim is to study legal norms. However, the path linking ought to do with heteronomous imperatives, offers another angle of attack to the notion of relevance of relevant logicians, rooted in the contentual approach of fully interpreted languages in the style of CTT. Indeed; according to our full formalization of an obligatory statement, pieces of evidence (proof-objects) are part of the content of the legal norm. Let us recall this formalization:[39]

$$[\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset R_1(y)\,] \wedge [\,(\forall z : \neg A_1)\,\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset S_1(z)\,]$$
$$\mathbf{true}\,(x : A_1 \vee \neg A_1)$$

or as an operator

[37] Cf. [20]. For a recent discussion on the issue see Hilpinen and McNamara [38, pp. 59–69; 97–110].

[38] However, this does not mean that a deontic notion of logical validity makes no sense at all. On the contrary, as pointed out by Martin-Löf [61], dialogical logic shows how to develop a conception of logic where deontic force constitutes meaning — see too [70, chapter 11.5].

[39] In the present paper we leave E out, since this relates to defeasibility, an issue that is linked to the dynamics of argumentation, which we deal elsewhere — Rahman and Iqbal [67] discuss defeasibility in the context of legal argumentation in Islamic Law, though they do not combine with an analysis of the deontic qualifications of actions.

$(\forall x : A_1 \vee \neg A_1)\,\{\,[\,(\forall y : A_1)\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset R_1(y)\,] \wedge$
$[\,(\forall z : \neg A_1)\,\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset S_1(z)\,]\,\}\,\mathbf{true}$

As mentioned in the introduction, in a more general framework, we can substitute *reward* and *sanction* by qualifying performances as being *law-abiding*, *law-breaking* and *legally neutral* (neither *law-abiding* nor *law-breaking*, or simply *not-sanctioned*) on the grounds of which the agent of those actions can be sanctioned or not — or within a value-approach to Law: *legally worthy, legally unworthy, legally neutral.*[40] Let us set this explicitly

- In the following we take in that "$R(x)$" stands for the qualification *law-abiding*, whereas "$S(x)$" for the qualification *law-breaking*.

  Legal Relevance: Notice that, as pointed out above, these qualifications are defined on the elements of the set of action-propositions. So the qualifications law-abiding and law-breaking results from the formation rules that set their meaning:

  $R(x) : prop\,(x : A)$
  $S(y) : prop\,(y : \neg A)$

In words, according to these formation rules, whereas evidences for individual performances of actions of the type (of the action-proposition) A are qualified as *law-abiding*, evidence for individual actions of the type (of the action-proposition) $\neg A$ are qualified as *law-breaking*. In fact, one might say that the formation rules establish the *legal relevance* of $R(x)$ and $S(y)$, in the sense that the formation rules set the specificity of these qualifications to a fixed type of action-propositions.

Moreover, since the imperative is a hypothetical, its conditions are not known to be verified: this is what assures its normative feature: legal norms are prescriptions to act after all. This means that, as mentioned above, no tautology can be subject of actions that qualify as law-abiding. Dually, contradictions can neither be subject of actions that qualify as breaking the law. Summing up: while the hypothetical feature assures that the conditions of the heteronomous imperatives at issue are logically contingent, the formation rules ensure their specificity for a fixed type of actions. This is what legal prescriptions and interdictions are about and this is the reason too why many of the main motivations of standard relevance logic do not apply.

What about the consequences of an obligatory action? Are they obligatory too? The analysis of either logical consequences or entailments of obligatory actions follow the hypothetical and formation restriction mentioned above. For the latter, take it that it is enforced by law that parents care of their children. This seems to entail that the obligation extends to both of the parents. If that is the case, the formation *rule* of parent, provides the answer, if the obligation only applies to one of both, the distribution of $R(x)$ and $S(x)$ establishes who of the parents carries the obligation.

A particular case of the logical entailment of obligations is the one where some knowledge operator is embedded within an obligation. Let us discuss it briefly.

[40] Hilpinen and McNamara [38, pp. 9–15] provide a survey of the axiological approach to normative concepts initiated by Alexius Meinong [62].

2.5.2 Åqvist's Paradox of Epistemic Obligation

Consider the following main steps in Åqvist's paradox of epistemic obligation.[41]

1. It is obligatory that Mr Jones knows A (i.e. that the bank is being robbed).
2. Mr. Jones's knowing A implies A.
3. Mr. Jones knows that the bank is being robbed.
4. It is obligatory that the bank is being robbed.

Premise two is a usual axiom within the logic for propositional knowledge. It is not clear that such a statement makes sense in the context of CTT but for the sake of the argument let take it as a true implication.

Now, in our setting; from the fact that Mr. Jones's knowledge is qualified as law-abiding, we cannot infer that the action of robbing the bank is also qualifies as law-abiding, despite the fact that we take it that knowing A implies A. The best we can infer from Mr. Jones's knowledge of A, is the conjunction of $R(k)$ and A (whereby "k" stands for the piece of evidence supporting Mr. Jones's knowledge of A). However this conclusion does not yield a paradox.

Moreover logical consequences of the condition can induce no harm by weakening the antecedent or the consequent of the implications involved in the hypothetical. Let see discuss this with some detail.

2.6 Logical Consequences, Ross's Paradox and it's Dual

2.6.1 Ross's Paradox

Ross's [73] paradox becomes harmless in the logic of deontic imperatives. The paradigmatic example of this paradox is based on the fact that we can infer it is obligatory to send a letter, or to burn it, from the premise that it is, say, legally obligatory to send it. But in the framework of heteronomous imperatives this paradox does not arise.

If action of type A has been chosen to be performed (send the letter), and we know that this performance is law-abiding (and we also know that omitting to perform it is law-breaking), weakening the antecedent is harmless.

If carrying out some action A is law-abiding, adding the performance of an action of type B (burning the letter) does not support the inference that performing B will also be law-abiding. More precisely $R(y)$ will still apply to the performance of A. Notice that weakening the consequent is harmless too: extending the consequent with a disjunction does not change the fact that the scope of $R(y)$ is still some performance of A.

For short Ross puzzle does not apply since whatever performance makes true *Sending the letter*, it is of a different type of the one that makes true *Burning the letter*. Similar applies to its dual based on conjunction.

[41] [9]

2.6.2 The Good Samaritan Paradox

Assume the following simplified version of Prior's [66] Good Samaritan Paradox.

1. It is obligatory that (Jones helps Smith and Smith is being mugged) ($\mathbf{O}(H \wedge M)$). By distribution of the operator and elimination of the conjunction we obtain the oddity:
2. Smith being mugged is obligatory ($\mathbf{O}M$)).

In our framework (2) cannot be inferred: if the conjunction qualifies as law-abiding, this qualification cannot be transferred to the qualification of the performance of each of the components of the conjunction.

Moreover, in our reconstruction of heteronomous imperatives the so-called *inheritance principle* from standard deontic logic[42] does not generally hold. According to this principle, if it is a theorem that A implies B, we can infer that this implication is obligatory. But in our framework logical inference does not influence the distribution of the legal qualifications $R(y)$ and $S(y)$. Something similar applies to the *equivalence rule*:[43] if it is a theorem that a certain bi-implication holds, according to our reconstruction, this does not have bearings on the distribution of the legal qualification involving that bi-implication.

On our view, the embedding of obligations can have sense, but only in contexts where the qualifications law-abiding and law-breaking are structured by dependences. For instance, it might be the case performing action B qualifies as law-abiding if that performance is made dependent upon performing action A, too.

More generally; the distribution of obligations over implications constitutes one of the oldest and most important puzzles of standard deontic logic and is known as Chisholm's paradox. As discussed in the following section Chisholm's puzzle provides some interesting insights on logic of norms, in the context of legal reasoning the setting is understood as establishing hypothetically the conditions for a given obligation, and at the same time assuming that there is a **performance** contrary to the duty expressed by that obligation.

2.7 Heteronomous Imperatives and Chisholm Puzzle in Legal Contexts

There is a wide agreement among experts in deontic logic that the following paradox cannot be solved with the means of standard deontic logic, because there is no satisfactory way in such a framework to combine obligation and implications. The original formulation of Chisholm [23] is based on the following two sets of sentences, such that each set is logically independent of the other

1. It ought to be that Jones goes to the assistance of his neighbours.
2. It ought to be that if Mr. Jones goes to the assistance of his neighbours, then he tells them he is coming.

[42] See Hilpinen and McNamara [38, p 38].
[43] Ibid.

3. If Mr. Jones doesn't go to the assistance of his neighbours, then he ought not to tell them he is coming.
4. Mr. Jones does not go to their assistance.

As pointed out by Hilpinen and McNamara [38, p. 83], it is widely accepted that the four sentences constitute a mutually consistent and logically independent set of sentences: all four might be ***true*** at once, and none is a deductive consequence of the others.

The problem for standard deontic logic, in a nutshell is that if we formalize the first pair of sentences as the set $\mathbf{O}A, \mathbf{O}(A \supset B)$, we obtain by the usual manipulations in basic modal logic the conclusion

Mr. Jones ought to tell to his neighbours that he is coming (i.e., $\mathbf{O}B$*),*

whereas from the second pair, if formalized as the set $\neg A \supset \mathbf{O}\neg B, \neg A$, we infer:

Mr. Jones ought not to tell his neighbours that he is coming (i.e., $\mathbf{O}\neg B$*),*

and this is **contrary to duty** in relation to the first pair.

One immediate reaction is to contest the ***narrow-scope*** formalization $\neg A \supset \mathbf{O}\neg B$ of (3) pointing out that its logical form is $\mathbf{O}(\neg A \supset \neg B))$, which renders the ***broad-scope*** of $\mathbf{O}$. However, because of the usual problems with material implication, such an approach leads to infer $\mathbf{O}(\neg A \supset \neg B)$ from $\mathbf{O}A$ — for a thorough discussion on the paradox and a survey of the main contemporary responses to it see Hilpinen and McNamara (2013 pp. 83-87).

In the following we will focus on both the broad and the narrow reading of sentence three.

2.8 Hypotheticals and Conditional Obligation.

According to our reading, the very point of a conditional obligation is that the distribution of law-breaking and law-abiding of a performance is now defined in the **context** of the performance of another one. In other words, we read the conditional obligation $C(A, B)$ as

- "It ought to be that if A is performed be means of action x, then $B(x)$ must also be performed".

It is this dependent structure of the conditional obligation, that allows qualifying the whole $C(A, B)$ as law-breaking or law-abiding. [44]

[44] Hypothetical imperatives underlie also Leibniz's approach to legal notion of *conditional right*. Indeed, in his work on Roman Law Leibniz pointed out, that conditional obligations as occurring in conditional-right should be understood as dependences within the hypothetical structure of what he calls *moral judgements*. For example, if the norm establishes that some *beneficiary* has the right to claim B from a *benefactor*, such as some payment agreed by a legally valid contract, under the condition A, then if the condition is satisfied the claim is due, but if the condition is not satisfied the consequent either. See Leibniz The *Disputatio Juridica (prior) De Conditionibus of* 1665, in Leibniz [54, A VI, I, pp. 97–150]; Armgardt [10, 11, 12, 13, 56, 57]. Since the work by Leibniz, legal norms in general are understood as having a conditional structure, particularly so in the Civil Law — cf. [63, p. 125].

In the case of Chisholm's puzzle, the conditional obligation of the second premise is not composed by joining independent obligations with an implication, but rather by the dependency of performing $B(x)$ upon performing A, namely, by Mr. Jones telling his neighbours about coming for assistance (his performing A).

The distributions of the deontic qualifications involved in the first two premises of Chisholm's puzzle are defined in the following way:

$R_A(x) : prop\,(x : A)$	Performing A qualifies as law-abiding (or rewarding performance).
$S_A(z) : prop\,(z : \neg A)$	Omitting to perform A qualifies as law-breaking (or sanctioning performance).
$R_B(x, y) : prop\,(x : A, y : B(x))$	Performing B in the context of a performance of A qualifies as law-abiding
$S_B(x, u) : prop\,(x : A, u : \neg B(x))$	Omitting to perform B in the context of a performance of A qualifies as law-breaking[45]

These yield

1. **It ought to be that** A
   Performing A qualifies as law-abiding and its omission qualifies as law-breaking. $(\forall x : A \vee \neg A)\,\{\,(\forall y : A)\,[\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset R_A(y)\,] \wedge [\,(\forall z : \neg A)\,\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset S_A(z)\,]\,\}\,\mathbf{true}$

2. **It ought to be that if** A, **then** B
   Performing B in the context of a performance of the condition A qualifies as law-abiding, and its omission qualifies as law-breaking. The omission of A qualifies as law-breaking. $(\forall x : A \vee \neg A)\,\{\,(\forall y : A)\,[\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset (\forall w : B \vee \neg B)\,\{\,(\forall u : B)\,[\,\mathbf{left}^{\vee}(u) =_{\{H2\}} w \supset R_A(y) \wedge R_B(y, u)\,] \wedge (\forall v : \neg B)\,[\,\mathbf{right}^{\vee}(v) =_{\{H2\}} w \supset S_B(y, v)\,]\,\}\,] \wedge (\forall z : \neg A)\,[\,\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset S_A(z)\,]\,\}\,\mathbf{true}$

Now, if we read "telling the neighbours about coming to assist them" in the second premise as a dependent action but we provide a narrow scope to the third premise, i.e.,

> *If Mr. Jones does not come for assistance, then it ought to be that Mr. Jones does not tell the neighbours he is coming for assistance;*

then "telling the neighbours about coming to assist them" is defined as being an action independent of A. So there is no puzzle, but rather an ambiguous reading of "B"; once as the dependent action $B(x)$: prop $(x : A)$ (premise 2) and then as the independent action $B : prop$ (premise 3).

If we wish to avoid this form of ambiguity, we might provide a uniform narrow scope reading to every obligation right from the start. Accordingly, the obligation to do B

[47] Such kind of clauses can be understood as *nullifications* — i.e. those clauses where omitting to perform B in presence of a performance of A is qualified as law-breaking, despite the fact that that the latter qualifies as law-abiding. In such kind of situations the performance of B ***abrogates*** the law-abiding performance of the condition. Consider the case of some who helps a verbally abused person, but finishes by injuring the abuser.

is to be formulated explicitly as independent in the second premise — the conditional obligation of the second premise is thereafter formulated as an implication that joins two independent obligations). But then it is obvious from the initial setting that the second and the third premise are in conflict — B is *obligatory* by premise (2), $\neg B$ is *obligatory* by premise (3). However, this contravenes the original agreement on the consistency of the (informal formulation of) the premises constituting the puzzle.

Alternatively, we might try to avoid ambiguous formulations by conceiving the consequent of the implication in the third premise, as involving the same conditional obligation of premise three. However, after detaching the consequent (with help of last premise $a : \neg A$) of premise three and joining the result with premise four, the following (quite odd) reading comes out

1. *It ought to be that If Mr. Jones does not come for assistance, he does not tell his neighbours he is doing so, since there is not performance of A to tell about* (recall that B has been defined as dependent upon A).

A further broad reading of the third premise involves a different norm. It might be formulated as a kind of further specification of the second premise (perhaps constituting a new more complex norm). So, while the second premise establishes how "*telling the neighbours about coming to assist them*" must be qualified in the context of a performance of A, the third premise might be understood as establishing how to qualify "*telling the neighbours about coming to assist them*" in the context of the absence of a performance of A. But from this it just follows that a performance of "*telling the neighbours about coming to assist them*" is qualified as law-abiding when performed in the context of A, and as law-breaking in the context of the absence of a performance of A — in the latter case Mr. Jones is just lying when he tells the neighbours that he is coming. So again, in fact we have a different kind of action, $B^{'}$ instead of B, an action that depends on $\neg A$ rather than on A.

$R_A(x) : prop\,(x : A)$	Performing A qualifies as law-abiding.
$S_A(z) : prop\,(z : \neg A)$	Omitting to perform A qualifies as law-breaking.
$R_B(z, w) : prop\,(z : \neg A, w : \neg B'(z))$	Omitting to perform $B^{'}$ in the context of an omission of A qualifies as law-abiding[46]
$S_B(x, v) : prop\,(z : \neg A, v : B^{'}(z))$	Performing $B^{'}$ in the context of an omission of A qualifies as law-breaking.[47]

In summary, according to our analysis of this situation, combining the right conjunct of the first premise, viz., $(\forall z : \neg A)\mathbf{right}^{\vee}(z) =_{H1} x \supset S_A(z)$ with the last premise

[46] Such kind of clauses can be conceived as *mitigations* — i.e. those clauses where omitting to perform B in context of absence of A is qualified as law-abiding, despite the fact that that the latter qualifies as law-breaking. In such kind of situations omitting to perform B ***mitigates*** the law-abiding performance of the condition. Consider the case of someone who after carrying out a law-breaking action, confesses voluntarily the delict.

[49] Such kind of clauses can be conceived as *aggravations* — i.e. those clauses where performing B in context of absence of A is qualified as law-breaking. In such kind of situations performing B ***aggravates*** the law-breaking performance of the condition. Consider the case of someone who after carrying out a delict commits perjury.

$a : \neg A$, gives $S(a)$, i.e., that Mr. Jones not going for assistance qualifies as law-breaking; furthermore, by (3), if he doesn't tell his neighbours that he is coming, this qualifies as law-abiding (or, rather, this is considered as a mitigating circumstance) and if he does lie about coming (by telling that he is coming for assistance whereas he is not), this qualifies as a law-breaking performance that aggravates the fact that Mr. Jones broke the law by not assisting his neighbours.

Perhaps this is the reading at stake in the original informal setting of the puzzle after all, or at least in those readings of the puzzle that see the third premise as a kind of updating the second premise.[48]

Let us finish this section by mentioning the point that it seems to be worthy studying Chisholm's puzzle in the context of Lemmon's (1962) distinction between duties (*duty of being a good citizen, a good neighbour*, and so on) and obligations (related to some committal actions like *promising, telling to come*, .etc.). This will be left for further work.

2.9 Conditional Obligation, Conditional Right

Within Civil Law conditional right constitutes one of the most important forms of conditional obligation. The paradigmatic example of conditional right of Roman Law establishes that *Secundus*, the *beneficiary*, has the right to claim B (the payment of a fixed sum of money) from *Primus*, the *benefactor*, under the proviso that the condition A (that a ship arrives from Asia) has been satisfied. If the condition (there is evidence for the arrival of a ship from Asia) is satisfied the claim is due, (Primus must pay) but if the condition is not satisfied the consequent either (Secundus cannot force Primus to pay). Condition and Conditioned of a conditional right are called *fact* and *jus* respectively.

According to our analysis, if the condition is satisfied, then the payment of the benefactor to the beneficiary qualifies as abiding the legal terms of the contract, whereas the non-payment qualifies as breaking those legal terms. If the condition is known not to be satisfied, then it is the non-payment that qualifies as abiding the legal terms of the contract (the *jus* is declared to be not legally enforceable) — see [11, 57] — this seems to have the logical structure of what we called *mitigation*. However, the most accurate way to deal with the case when the condition is not satisfied is to define a new predicate

[48] It seems that cases of what has been called *pragmatic oddity*, can be analysed as a special form of conditional obligations - cf. Prakken and Sergot (1997) and Parent and Van der Torre (2017). Indeed, assume the following:

1. It as obligatory to keep promises.
2. It is obligatory to apologize for not keeping them (those promises in (1)).

If we formulate (2) as a conditional obligation in a similar way as we did with the second premise of Chisholm's puzzle, then the distribution of the deontic qualifications law-abiding and law-breaking for the act of apologizing will be defined in the context of not kept promises. The point again is that the qualification for keeping a promise is different from the one that qualifies deontically apologies for not keeping it. Developing a detailed analysis of the case of pragmatic oddity will be discussed in a follow up paper.

"$NS(x, y)$" to be read as "instances of not paying do not qualify as law-breaking (or paying is not legally enforceable)":

$$\{\,(\forall y : A)\,[\,\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset (\forall w : B \vee \neg B)\,\{\,(\forall u : B)\,[\,\mathbf{left}^{\vee}(u) =_{\{H2\}} w \supset R_B(y, u)\,]\,\wedge\,(\forall v : \neg B)\,[\,\mathbf{right}^{\vee}(v) =_{\{H2\}} w \supset S_B(y, v)\,]\,\}\,]\,\wedge\,(\forall z : \neg A)\,(\forall n : \neg B)\,[\,\mathbf{right}^{\vee}(z) =_{\{H1\}} x \supset NS_B(z, n)\,]\,\}\,\mathbf{true}\,(x : A \vee \neg A)$$

If the condition is known not to be satisfied, but the payment takes place anyway this payment qualifies as being beyond the legal terms established by that specific contract, however it does not qualify as breaking the legal terms of the contract either. The payment will then count as being a *donation* rather than the effect of the conditional right established by the contract at stake (a donation might also be established by a contract of its own).[49]

The lesson to be drawn from this example is that within legal contexts, the qualifications law-abiding and law-breaking might take forms specific to the legal validity of the norm under consideration, such as conditional right, donation, statutory right and so on. Working out such distinctions is a task we do not undertake here.

2.10 Permissibility, Iteration and the Dynamics of Legal Systems

As mentioned above, the general principle of Islamic jurisprudence

All actions are permissible unless proscribed by Law,

required the deployment of a dialectical system of argumentation called *qiyās* that regulated the integration into the legal system of an explicit updated deontic qualification (possibly different to *permissible*) for a new kind of actions.

In fact, such a kind of principle makes it possible that the system updates itself constantly in the way Walter E. Young (2017) calls the *dialectical forge*: without it the legal system remains closed and static.

Moreover, in this context, iterations such as

It ought to be that A is obligatory

Is not to be read as the sheer iteration of the deontic operator but as the call to integrate into the legal system the obligation of A. In other words,

It ought to be that A is obligatory

reads

The norm "A is obligatory" should be integrated into the legal system

And this is again calls for a legal argumentation, justifying the claim. Similar for

It ought to be that A is reprehensible,
It ought to be that A is recommendable,

[49] Mathias Armgardt elucidated this point to Rahman in a personal email.

3 Conclusions and Work Ahead: On The Rationality of Law.

John Woods's book *Is Legal Reasoning Irrational. An Introduction to the Epistemology of Law*, stresses the point that the rationality of Law stems from its common-sense agent-driven practice rather than from an alleged underlying sophisticated logical structure. Accordingly he accurately points out many of the blind-spots and short comings of standard logical analysis, when applied to legal reasoning.

The main aims of the present paper, are twofolds, on one hand we show that the Arabic Jurisprudents of the Middle Ages, driven by the practical needs of applying Law to the new cultural contexts required by the expansion of the Islamic world, developed and defended a rational concept of Law based on a logical and content-based analysis of legal normativity and its implementation for actual cases. On the other, we suggest that a suitable generalization and reconstruction of the Arabic framework provides some useful tools for the logical analysis of deontic qualifications within contemporary legal reasoning.

Besides the patent fact that we skip many important issues discussed in Woods [80], we concede that the enterprise is incomplete also in another important aspect. Indeed, in order to deliver an overall picture of our approach we need still to combine the present analysis with recent studies in legal argumentation within Islamic Law.[50] Particularly so in relation to defeasibility, studied by Rahman and Iqbal [67] in the context of parallel-reasoning, this is still subject of work in progress.

The present framework deploys some technical logical devices that might be received with scepticism by John. The general point of it is that the rationality of Law emerges from delving into the conceptual structure of their norms. This does not mean that we assume that in the court the lawyer needs to carry out a complete formalization of the argument at stake. However, we think that awareness of the logical structure of norms and their meaning explanations provides the lawyer with some tools for a fine-grain analysis of the legal decision to be taken. This is, we suggest, a wise lesson of the Elders we should not ignore.

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[50] See, Rahman and Iqbal [67, 68].

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